Evaluation of meat-hook handle shapes

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Abstract

Seven meat hooks including two current designs and five new designs with flat-rectangular, frustum, double frustum, and cylindrical shapes were evaluated. A two-phase study was conducted. In the first experiment, maximum pulling forces were measured by load cell to evaluate the effects of handle shapes on subjective discomfort, maximum pulling force, and muscle activity. Two pulling forces, 15 and 30 kg, were employed to a pulley mechanism to simulate pulling a beef carcass horizontally in the second experiment. FSR (force sensitive resistor) glove was used to measure the pulling forces on the meat hooks. The glove has 12 sensors that result in placement on the pulpy regions of each phalange. In addition, a biomechanical hand model was developed and applied to predict tendon forces. Double frustum shaped handles produced significantly larger maximum pulling forces and best force efficiency when normalizing forces with EMG. In terms of external forces as measured by the FSRs, the averages of finger force contributions to the total finger force were 27%, 32%, 32%, and 10% in order from index finger to little finger. The averages of phalange force contributions to the total finger force were 20.9%, 33.7% and 45.4% for the distal, middle and proximal phalanges, respectively. A Chi-square analysis indicated that the phalange force distribution for double frustum handles deviated least from the average contributions for all hooks. Double frustum handles showed the least predicted tendon forces and normalized tendon forces per unit external force. The optimality of double frustum shaped handles was also supported by the lowest discomfort ratings. Therefore, based on both empirical physiological measurements and theoretical biomechanical calculations, a double frustum handle is most efficient for pulling task, producing the least amount of tendon forces.

Relevance to Industry

Results obtained from this handle study will help to reduce the ergonomic risk factors of finger/hand disorders for meat hook workers who use the meat hooks with a long daily duration and high force.

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1. Introduction

Poor design and excessive use of hand tools are associated with increased incidences of work-related disorders or cumulative trauma disorders...
(CTDs) of the finger, hand and forearm (Armstrong, 1983; Aghazadeh and Mital, 1987). Improperly designed hand tools lead the workers to high grip forces, awkward hand/wrist postures, and more fatigue. Therefore, the proper design and evaluation of hand tools are important issues in industry. To design better hand tools, several characteristics of hand tools such as handle size (Putz-Anderson, 1988), shape (Tichauer and Gage, 1977; Scheller, 1983; Cochran and Riley, 1986), surface type (Lewis, 1987; Fellows and Freivalds, 1991), and texture (Pheasant and O’Neill, 1975) have been considered.

Handle size is one important hand tool design factor to maximize grip strength, reduce stress on the digit flexor tendons, and avoid stresses to the first metacarpal ulnar collateral and carpometacarpal ligaments (Chaffin and Anderson, 1984). Khalil (1973) compared an elliptical, a spherical handle, and cylindrical handles and found that a cylindrical handle of 32 mm diameter was less integrated EMG than other handles in static torque tasks. Pheasant and O’Neill (1975) tested the commercially available screwdriver handles with smooth and knurled cylinders of the same diameter. Evaluation of handle shape on grip fatigue in manual lifting did not indicate any significant difference in shape when the effect of handle diameter was eliminated. Finger grooves designed to provide form-fitting handles are generally undesirable (Tichauer, 1973; Mital, 1991; Johnson, 1993). Although the finger grooves may look as though those were molded to the hand, those are only fit to one particular hand size. This may result in discomfort, nerve compression and impairment of circulation for a person with an exceptionally large or small hand (Meagher, 1987).

A better approach is to change the handle diameter according to the length of the fingers. Konz (1974) and Bullinger and Solf (1979) suggested a new handle design using a hexagonal cross section, instead of circular cross section, shaped as two truncated cones joined at the largest ends. The shape fits the contours of the palm and thumb best in precision and power grips, and produced higher torque than conventional handles (Freivalds, 1996).

Finger/phalange force contributions with various cylindrical handles have been studied by many researchers (An et al., 1978; Ejeskar and Ortengren, 1981; Amis, 1987; Lee and Rim, 1990; Radhakrishnan and Nagaravindra, 1993). They reported that the average percentage contributions of finger forces to the total grip force, from index to little fingers, were 28.3%, 31.0%, 23.2%, and 17.4%, respectively. They also found that the average contributions of distal, proximal, and middle phalanges to the total grasp force were 46.9%, 32.9%, and 20.2%, respectively.

Many researchers have studied tendon forces of the index finger in pinching actions (Smith et al., 1964; Chao et al., 1976; Berme et al., 1977; An et al., 1979; Weightman and Amis, 1982) using 2D or 3D biomechanical finger models. From the solutions of force/moment equilibrium equations, for an external load of \( P \), \( 2.1 - 4.3P \), \( 0.73 - 2.5P \) and \( 1.1 - 7.1P \) values were reported for the flexor digitorum profundus (FDP), flexor digitorum sublimis (FDS) and intrinsic tendons, respectively. Chao et al. (1976) applied their 3D finger model to the tip, lateral, ulnar pinching as well as grasp actions. They reported that the ranges of unit tendon forces of FDP, FDS and UI were \( 2.53 - 5.95P \), \( 3.05 - 4.23P \) and \( 3.37 - 5.2P \) for the index, middle and little fingers, respectively. These biomechanical models, however, focused on only understanding a force distribution of the fingers and were not used to evaluate hand tools.

The meat hook is a common hand tool in meat packing to pull heavy beef carcasses. Use of such hooks over long-time periods and with high forces may induce work-related disorders for the hand and fingers such as tendonitis in the ring finger.

The objective of this study was to evaluate the effect of handle shapes on maximum pulling task, muscle activity, finger force distribution, handle discomfort and predicted finger tendon force and to determine an optimal handle shape by use of flexor muscle activity and pulling force.

In this study ‘efficiency’ was defined as the ratio of pulling force to normalized flexor muscle activity. An ‘optimal handle’ is one which results in the maximum efficiency ratio.
2. Method

2.1. Meat hooks

Two meat hooks currently used in the meat-packing industry and five new designs were evaluated. Hook A, a current model, is a 15 mm thickness piece of flat polyethylene with a hook inserted in the center of the piece (termed center), and Hook B, also a current model, is a polyethylene frustum. The cross-sectional shape of Hook B is a circular and diameters of the handle were gradually decreased from bottom (35 mm) to top (30 mm) with the hook inserted off-center, whereas Hook C has a circular cross-sectional shape and diameters were gradually increased from bottom (30 mm) to top (35 mm) with the hook inserted off-center. The hooks of handle D and E are placed at the center and off-center of the handle, respectively. Both hooks are double frustum shapes, which are the diameters of the handle gradually decreased from the center (38 mm) to both ends (26 mm). Hook F and G are cylindrical shapes with the hook off-centered and centered, respectively (Fig. 1). Note that for off-center handles, the hook projects between the index and middle fingers, while for the center handles, the hook projects between the middle and ring fingers.

2.2. Measurement system

Silver–silver chloride surface electrodes were used to record the electrical activity of the flexor digitorum superficial muscle under the maximum pulling task. The EMG signal was acquired using the FlexComp system (NexGen Ergonomics, Montreal, Canada). The signal was recorded and then was amplified, rectified, and filtered using a low-pass filter with 60 Hz cut-off frequency. For each test condition, the EMG data were collected for a period of 5 s at a rate of 496 samples/s.

A force glove was developed by overlaying force-sensing resistors (FSRs) on a glove to measure finger forces. Twelve FSRs were placed on the pulpyparts of each phalange (Fig. 2). The active area of the FSR sensor is 5 mm diameter and 0.3 mm thickness. The output signals from the FSR were sent to a custom-made voltage division circuit box which was designed to provide 0 to ±5 V DC to the A/D converter (DASH 16-F METRABYTE, 1989). A data acquisition software system was also developed.

A biomechanical hand model was developed based on the assumption of a constant moment arm of the tendons at the finger joints. In case of constant tendon moment arms, Landsmeer’s (1962) first tendon model was used to express the relationship between tendon excursion and joint angles. Equilibrium equations were derived using the principles of virtual work model (Storace and Wolf, 1979). Three tendon forces (flexor digitorum...
profundus, FDP; flexor digitorum sublimes, FDS; ulnar interosseous, UI) were derived from the equilibrium equations (Appendix A).

2.3. Experimental design

2.3.1. Experiment 1: Maximum pulling force

A total of 6 subjects were recruited from the Pennsylvania State University to take part in this study. The subjects were all male, aged 23–34. All subjects were screened by questionnaire for any hand and wrist injuries or surgeries, which may limit their physical activities. Subjects were provided with a brief description of the goals and procedures. Then, they signed an informed consent form prior to experimentation.

The subject was asked to use his left hand to pull the load cell equipment for 3 s with his maximum voluntary contraction. All seven hooks were tested for each subject. A handle of the load cell was attached to the 1.5 m height custom-made frame. The average maximum pulling force during task was shown on the digital screen of the load cell equipment. EMG signals were collected from the FlexComp system. To eliminate any effects of background noise and minimize inter-subject and inter-electrode differences, the EMG was normalized (EMGN) as follows: EMGN = (task EMG – rest EMG)/(maximum EMG – rest EMG).

"Efficiency" is defined as the ratio between maximum pulling force and flexor muscle activity measured (Ayoub and Lo Presti, 1971), used to determine the optimal hook handle. The subjective ratings of handle discomfort (Borg, 1973) were also recorded for each hook handle. The scale ranges were from 6 (very very light) to 20 (very very hard). Each subject performed four repetitions for each hook with 3 min resting time between each trial. All 28 trials (7 hooks × 4 trials) were tested in a completely randomized order.

Subjects and trials were considered random factors. Dependent variables were maximum pulling force, flexor muscle activity, and subjective ratings of discomfort. The ANOVA was performed to evaluate all possible effect of handle type on the dependent variables.

2.3.2. Experiment 2: Finger force distribution

Four different males participated in the study. They were all right-handed university students, ranging in age from 25 to 30. All subjects were screened by questionnaire for any hand and wrist injuries or surgeries, which may limit their physical activities. Subjects were provided with a brief description of the goals and procedures and signed an informed consent form.

The subjects used the force glove to evaluate finger and phalange force distributions. Two different hanging weights, 15 and 30 kg, were employed. Pulling these weights with hooks, acting horizontally through a pulley system, was comparable to average and maximum forces for pulling a beef carcass at a meat packing company (Fig. 3). Three trials were collected for each condition. A total of 42 trials (2 weight × 7 hooks × 3 trials) were completely randomized.

Dependent variables were finger/phalange forces during pulling, flexor muscle activity, and subjective ratings of discomfort. The effect of handle type, finger, phalange, and weight on the dependent variables was analyzed.

3. Results

3.1. Maximum pulling force

Maximum pulling forces on the hook handles differed significantly at the α = 0.05 level, with Hooks D and E (double frustum handles) showing significantly higher forces as compared with other hook handles (F6,153 = 5.89, p<0.001). Fig. 4 shows the average maximum pulling force for each hook handle.

Electromyography (EMG) was also analyzed to objectively determine the optimal hook handle shape for maximum pulling force. Fig. 5 presents the results of the efficiency for each hook. Although the results of ANOVA showed that no statistically significant difference between hook handles, Hooks D and E showed higher efficiencies than the other hook handles.

Analysis of subjective discomfort ratings showed that Hooks D and E (double frustum handles) and B (oval handle) had ratings of 9.8,
10.8 and 10.8 (*very light to fairly light*) whereas, Hook A (flat) showed a 16.7 (*very hard*) for the maximum pulling task. Tukey test results are shown in Table 1. Mean subjective ratings grouped by vertical lines do not differ significantly at $\alpha = 0.05$. Subjects preferred double-frustum shapes over other shape handles.

3.2. Force distribution

The ANOVA of the pulling force for two different weights (Table 2) shows a statistically significant effect for all main effects except hooks. Although the effect of hooks is not statistically significant, Hook D and E generally required lower finger forces than the others.
The mean finger forces were significantly different \((F_{3,156} = 3.61, \ p<0.013)\) and, in descending order by magnitude of force, are middle, ring, index and little finger. Total pulling force was assessed by summing the individual sensors and used to compute the average contribution of individual finger forces. The average contribution, in order from index finger to little finger was 27\% (ranges: 22.2–30.4\%), 32\% (ranges: 27.9–34.0\%), 32\% (ranges: 30.4–34.2\%) and 10\% (ranges: 9.2–11.2\%), respectively.

The contribution of individual finger forces to the total pulling force and Chi-square test of average finger contribution for all hooks are shown in Fig. 6.

A Chi-square analysis yielded values of 0.87, 0.58, 1.87, 1.05, 1.51, 1.51, and 3.03 for Hooks A through G, respectively. Although these values were significantly lower than the critical value of Chi-square \(\chi^2 (0.95, \ 2)=5.99\), the phalange force contribution of Hook D is the most evenly distributed over each phalange, whereas Hook A has very uneven distribution over each phalange in the pulling task.

For all seven hooks, the proximal phalanges exerted the greatest force, followed by the middle

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Table 2
ANOVA on finger force

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>3</td>
<td>60.512</td>
<td>20.272</td>
<td>20.65</td>
<td>&lt;0.001*</td>
</tr>
<tr>
<td>Hooks</td>
<td>6</td>
<td>7.063</td>
<td>1.177</td>
<td>1.20</td>
<td>0.304</td>
</tr>
<tr>
<td>Weight</td>
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<td>7.792</td>
<td>7.792</td>
<td>7.94</td>
<td>0.005*</td>
</tr>
<tr>
<td>Finger</td>
<td>3</td>
<td>27.788</td>
<td>3.542</td>
<td>3.61</td>
<td>0.013*</td>
</tr>
<tr>
<td>Phalange</td>
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<td>72.973</td>
<td>36.486</td>
<td>37.17</td>
<td>&lt;0.001*</td>
</tr>
<tr>
<td>Trial</td>
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<td>2.020</td>
<td>1.010</td>
<td>1.03</td>
<td>0.358</td>
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<tr>
<td>Error</td>
<td>1566</td>
<td>1537.2</td>
<td>0.982</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1583</td>
<td>1715.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p<0.05.

The mean forces of the phalanges are also significantly different \((F_{2,56} = 37.17, \ p<0.001)\). The forces produced by the proximal phalanges were the greatest followed by middle and distal phalanges. The distributions of phalange force to the total pulling force are shown in Fig. 7. A Chi-square analysis for the average of all hooks yield values of 3.76, 0.62, 0.67, 0.48, 0.77, 1.23, and 1.12 for Hooks A through G, respectively. Although these values are significantly lower than the critical value of Chi-square \(\chi^2 (0.95, \ 2)=5.99\), the phalange force contribution of Hook D is the most evenly distributed over each phalange, whereas Hook A has very uneven distribution over each phalange in the pulling task.

For all seven hooks, the proximal phalanges exerted the greatest force, followed by the middle
phalanges and the distal phalanges. That is, an average 45% (ranges: 43.8–46.6%) of total finger force was exerted by the proximal phalanges, 34% (ranges: 31.8–37.4%) by the middle phalanges and 21% (ranges: 16.3–22.5%) by the distal phalanges, respectively.

3.3. Biomechanical hand model

A biomechanical hand model was developed based on the principles of virtual work and applied to evaluate hook handles (Appendix A). Basically, the empirical FSR forces for each hook were used in the biomechanical hand model to predict tendon forces of flexor digitorum profundus (FDP), flexor digitorum sublimis (FDS) and ulnar interosseous (UI) during pulling tasks. Figs. 8 and 9 depict that the average total tendon forces and normalized tendon forces per unit force, respectively.

Hooks D and E (double frustum handles) showed the least tendon forces for the total pulling forces as well as the unit external forces. Hook A had the highest tendon and normalized tendon forces. The ranges of tendon forces per unit external force ($P$) for all three tendons were $0.47–0.84P$, $0.69–1.10P$ and $1.45–2.54P$ for FDP, FDS and UI, respectively and overall tendon forces of the pulling tasks for all types of hooks were 3.15–3.7 times for the unit external force.

4. Discussion

The double-frustum shaped handles (Hook D and E) showed significantly larger maximum pulling forces than the other handles. The currently used flat rectangular handle (Hook A) had the lowest maximum pulling force. The ratio of maximum pulling force to normalized EMG activity was used to evaluate efficiency of hook handles and resulted in Hook E being best, followed by Hook D. In the second study, Hook D and E (double frustum handles) required less pulling force to maintain the given loads, whereas Hook F and G (cylindrical handles) required more force.

The subjective discomfort ratings of the double frustum handles were also better than other handles. Hook D provided the best subjective discomfort ratings among these handles followed by Hook E and B (9.8, 10.8 and 10.8, respectively). The double frustum shapes allowed each finger to grip at its optimum circumference. The longest middle finger had large circumference, whereas the shortest little finger had small circumference. It may make users feel more comfortable during a pulling task, if the handle size fit to users’ hand size. Most subjects complained of a pain to the proximal phalanges when they used Hook A during the pulling tasks. The relatively sharp edges of this flat handle (Hook A) dig into the palm near the proximal phalanges of the fingers. According to the Chi-square analyses, subjects generally preferred uniform phalange force distribution (i.e., 0.48, 0.77 and 0.62, for Hook D, E and B,
respectively) whereas, they did not have preference for more uniform finger force distribution (i.e., 0.87 for Hook A). Therefore, an optimal handle shape should have stronger fingers providing a larger contribution to total force than weaker fingers, with a uniform force distribution on all phalanges.

Finger and phalange force distributions for pulling tasks were studied using a force measurement system. The index finger contributes between 27.8% and 30.4% to the total pulling force, the middle finger 27.9–34.0%, the ring finger 30.4–34.2% and the little finger 7.2–11.2%. The contributions of index and middle fingers in the pulling tasks are in line with average values of 27.7% and 30.9% found in the literature whereas, the contributions of ring and little fingers do not correspond with average values of 23.5% and 17.9% found in the previous gripping studies (Amis, 1987; An et al., 1978; Chen, 1991; Ejeskar and Ortengren, 1981; Lee and Rim, 1990; Radhakrishnan and Nagaravindra, 1993). The sum of contributions of middle and ring fingers is 63.2% (ranges: 60.3–68.2%) in the pulling tasks, whereas the sum of contributions of middle and ring fingers is 54.4% in the cylindrical grip studies. That is, the role of ring finger in pulling tasks was significantly more important than in cylindrical grip actions. This may explain the high frequency of trigger finger injuries in the ring finger observed in meat packers. According to the injury records of one of the meat packing company, the ring finger is the most commonly affected digit, followed by the middle, index and little fingers in the jobs requiring use of meat hooks.

The average phalange force contributions to the total pulling force are 20.9%, 33.7% and 45.4% for the distal, middle and proximal phalanges, respectively. The high concentrations of the both middle and proximal phalanges (77.6–83.7%) may explain the frequency of trigger finger injuries in pulling tasks other than static holding or grasping tasks. According to the Karwowski and Salvendy (1998), the localized compression in an area of the A1 pulleys (between proximal phalanges and metacarpals) is one of main factors in trigger finger injuries. Flat rectangle handle (Hook A) showed high percentage of middle and proximal phalange forces whereas, double frustum handles showed a lower percentage of force contributions of these two phalanges in this study.

The biomechanical hand model predicted average unit tendon forces of FDP, FDS and UI were 0.7P, 0.94P and 1.74P in this study. According to the previous studies, the ranges of unit tendon forces of FDP, FDS and UI were 2.53–5.95P, 3.05–4.23P and 3.37–5.2P, respectively. Although these values are somewhat lower as compared with previous research, they still may be considered as a form of grip efficiency. The tendon forces predicted from the biomechanical hand model showed a good correlation with the results of other measures. According to the study, double frustum handles showed the least tendon forces, whereas Hook A showed the highest tendon forces. Therefore, double frustum handles would be recommended to make better subjective discomfort, performance (pulling force) and less finger tendon force in a pulling task.

Acknowledgements

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Appendix A. Formulation of hand model

A.1. Displacements of the flexor tendons (FDP, FDS and UI)

\[
\text{FDP} : X_P = -PR_1 \theta_1 - PR_2 \theta_2 - PR_3 \theta_3, \quad (A.1)
\]

\[
\text{FDS} : X_S = -SR_2 \theta_2 - SR_3 \theta_3, \quad (A.2)
\]

\[
\text{UI} : X_{UI} = -UR_3 \theta_3, \quad (A.3)
\]

where i is the 1, 2, 3, and 4: index, middle, ring and little fingers, \(X_P\) the FDP displacements, \(X_S\) the FDS displacements, \(X_{UI}\) the UI displacements, \(\theta_i\) the angle between middle and distal phalanx, \(\theta_2\) the angle between proximal and middle phalanx, \(\theta_3\) the angle between metacarpal and proximal phalanx, \(PR_i\) FDP moment arm at the joint i
where $\theta X$ is the increment of tendon excursion and $\partial \theta$ the increment of joint angle.

The virtual displacements are related by:

$$ \partial X_p = [(\partial X_p/\partial \theta_0) \partial \theta_0 + (\partial X_p/\partial \theta_2) \partial \theta_2 + (\partial X_p/\partial \theta_3) \partial \theta_3], $$

(A.5)

$$ \partial X_S = [(\partial X_S/\partial \theta_0) \partial \theta_0 + (\partial X_S/\partial \theta_2) \partial \theta_2 + (\partial X_S/\partial \theta_3) \partial \theta_3], $$

(A.6)

$$ \partial X_{UI} = [(\partial X_{UI}/\partial \theta_0) \partial \theta_0 + (\partial X_{UI}/\partial \theta_2) \partial \theta_2 + (\partial X_{UI}/\partial \theta_3) \partial \theta_3]. $$

(A.7)

Substituting $n, \partial X_S, \partial X_R$ and $\partial X_{UI}$ in calculate in Eqs. (A.5)-(A.7) into Eq. (A.4) become:

$$ F_p(\partial X_p/\partial \theta_0) = T_1, $$

(A.8)

$$ F_p(\partial X_p/\partial \theta_2) + F_S(\partial X_S/\partial \theta_0) = T_1 + T_2 $$

(A.9)

and

$$ F_p(\partial X_p/\partial \theta_3) + F_S(\partial X_S/\partial \theta_3) + F_{UI}(\partial X_{UI}/\partial \theta_3) = T_1 + T_2 + T_3. $$

(A.10)

For the linear displacement functions 1, 2 and 3, the equilibrium equations yield:

$$ PR_1 F_P = T_1, $$

(A.11)

$$ PR_2 F_P + SR_2 F_S = T_1 + T_2 $$

(A.12)

and

$$ PR_3 F_P + SR_3 F_S + UR_3 F_{UI} = T_1 + T_2 + T_3. $$

(A.13)

Therefore,

$$ F_P = T_1/PR_1, $$

(14)

$$ F_S = (T_1 + T_2 - PR_2 F_P)/SR_2, $$

(A.15)

$$ F_{UI} = (T_1 + T_2 + T_3 - PR_3 F_P - SR_3 F_S)/UR_3. $$

(A.16)

### A.2. Tendon moment equilibrium equations of index finger (DIP, PIP, & MP joint)

Three moment equilibrium equations can be obtained from each joint and calculated by the following equations:

$$ T_1 = F_1 L_{11}/2[\cos(-90 + \theta_1 + \theta_2 + \theta_3) \times \cos(-90 - \theta_1 - \theta_2 - \theta_3) \times \sin(-90 + \theta_1 + \theta_2 + \theta_3) \times \sin(-90 - \theta_1 - \theta_2 - \theta_3)] = F_1 L_{11}/2\cos(-180) = -F_1 L_{11}/2, $$

(A.17)

![Fig. 10. Free-body diagrams for finger and phalanges.](image-url)
\[ T_2 = F_1 \left[ L_{12} \sin(\theta_1 + \theta_2 + \theta_3) \cos(\theta_2 + \theta_3) \right. \\
+ L_{11}/2 \sin(\theta_1 + \theta_2 + \theta_3) \cos(\theta_1 + \theta_2 + \theta_3) \right. \\
- F_1 \left. \left[ -L_{12} \cos(\theta_1 + \theta_2 + \theta_3) \sin(\theta_2 + \theta_3) \right. \\
+ L_{11}/2 \cos(\theta_1 + \theta_2 + \theta_3) \sin(\theta_1 + \theta_2 + \theta_3) \right. \\
+ F_2 L_{12}/2 \sin(\theta_2 + \theta_3) \cos(\theta_2 + \theta_3) \\
+ \cos(\theta_2 + \theta_3) \sin(\theta_2 + \theta_3) \right. \\
= F_1 \left[ \left\{ L_{12} \sin(\theta_1 + \theta_2 + \theta_3) \cos(\theta_2 + \theta_3) \\
- \cos(\theta_1 + \theta_2 + \theta_3) \sin(\theta_1 + \theta_2 + \theta_3) \right. \\
+ L_{11}/2 \sin(\theta_1 + \theta_2 + \theta_3) \cos(\theta_1 + \theta_2 + \theta_3) \right. \\
+ F_2 L_{12}/2 \sin(\theta_2 + \theta_3) \cos(\theta_2 + \theta_3) \\
+ \cos(\theta_2 + \theta_3) \sin(\theta_2 + \theta_3) \right. \\
= F_1 \left[ \left\{ L_{12} \sin(\theta_2 + \theta_3) \sin(2\theta_1 + \theta_2 + \theta_3) \right. \\
+ F_2 L_{12}/2 \sin(2\theta_2 + 2\theta_3) \right. \\
+ L_{13} \sin(\theta_2 + \theta_3) \right. \\
+ F_3 \left. \left[ L_{13}/2 \sin(2\theta_3) \right. \\
\right], \tag{A.19} \]

where \( T_1, T_2, T_3 \) is the moments acting on the DIP, PIP, and MP, \( F_1 \) the external force acting vertically upward on the phalange (1 = distal; 2 = middle; 3 = proximal), \( L_{11}, L_{12}, L_{13} \): the length of the proximal, middle, and distal phalanges of index finger.

References


